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FREE-FALL TRAJECTORIES FROM THE MOON TO THE LAGRANGIAN EQUILIBRIUM POINTS; R. A. Broucke, Jet Propulsion Laboratory, Pasadena, California 91103.

In the last two years some interest has developed in the construction of large manned space stations. These stations would be orbiting in the Earth-Moon system and would be constructed and assembled in space from materials taken partly from the Earth, but mostly from the Moon. Several transportation systems could be considered. They range from magnetic accelerators located on the Moon to space-shuttle type vehicles traveling back and forth. Several possible locations for the large manned station have also been proposed, and the equilibrium points (Lagrange Points) are favorite candidates, especially the triangular points L_4 (60 degrees ahead of the Moon) and L_5 (60 degrees behind the Moon), or even the collinear points L_1 (in front of the Moon) or L_2 (behind the Moon).

We have thus undertaken a systematic study of the possible free-fall trajectories connecting the Moon with any of the equilibrium points $L_1,\ L_2,\ L_3$ and L_4 . We only consider here those free-fall trajectories which are of enough practical value: the trajects should be as direct as possible, with short transit times and with small velocity at the arrival at the Lagrangian point. As a model, we have taken, for the present initial exploration, the well known circular restricted three-body problem in two dimensions. The Earth and the Moon are the only two acting bodies; they are point-masses in circular orbits around each other. The true radius of the Moon does not enter the problem. A Levi-Civita regularization was used for the numerical integration of trajectories in proximity of the Moon.

We have used the symmetry properties (mirror image theorem) of the restricted problem in order to reduce the exploratory work: to each trajectory corresponds a mirror image on the other side of the Earth-Moon line (Syzygy-axis). For instance, to each Moon-to-L $_4$ trajectory corresponds a symmetric L $_5$ -to-Moon trajectory (traveled in the same time).

In the examples that are given below, we specify the trajectories by their final condition at L_1 , L_2 , L_3 , or L_4 rather than the initial conditions at the Moon. This can be reduced to a set of initial conditions at the Moon by integrating backwards or more simply by using the mirror image theorem. The parameters which are used to specify a given trajectory are the 2 coordinates of the libration point together with the velocity and flight path angle at the point. The flight path angle is measured with respect to the Earth-Moon line and is zero in the Earth-to-Moon direction (180 degrees in the Moon-to-Earth direction). These angles (and also the velocities) are relative quantities measured in the rotating Earth-Moon system.

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An example of the trajectory from L_5 to the Moon (or from the Moon to L_4) has been computed with a velocity of 90m/sec, a flight path angle of 205 degrees and a transit time of 15 days. This trajectory is mostly inside the Moon's orbit and is faster than the trajectories from L_4 to the Moon (or from the Moon to L_5) which are retrograde and mostly outside the Moon's circular orbit. Two such trajectories were found with a velocity of 90m/sec at L_4 . In the first one the transit time is 19 days and the angle at L_4 is 96 degrees, while for the second one these numbers are 206 degrees and 25 days.

Several similar trajectories have also been computed for the Lagrange points L_1 and L_2 . The point L_1 can be reached via a fairly direct path. The hidden point L_2 can be reached with a residual velocity of 230m/sec in 2.8 days. The flight-path angle at arrival at L_2 is 70 degrees.

Because so much work has been done on the study and classification of periodic orbits in the restricted three-body problem it is of interest to relate these trajectories to some of the known classical families of periodic orbits. It is found, for instance, that the above-mentioned trajectory from the moon to L_4 is very nearly the known periodic collision orbit of the retrograde family around the equilibrium point L_2 .

Inspection of the complete trajectory shows that it would be possible to take off from the Moon at the sub-Earth point, travel to L_4 , keep on going behind the Moon (intersect the syzygy-axis at the right angle at an apogee about 200,000 km behind the Moon) and then travel on to L_5 and finally return to the Moon and land at the departure point. Let us also mention that all the members of this family of periodic orbits have unstable characteristic components. Also, that this family corresponds to class a of Stromgren's early investigations.

The examples that we have elected to describe here seem to be some of the best compromises between fuel consumption and time of flight, but we intend to study more in detail the possibilities of other practical trajectories in the Earth-Moon system. Also, the different approach and landing paths on the Moon should be studied, as well as the sensitivities of the different trajectories.